3.4 derivatives → 3.5 chain rule

2014-10-17 day 37

Product Rule

$$\frac{d}{dx}(fg) = f'g + fg'$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{g^2}$$

Chain Rule (composition of functions)

$$\frac{d}{dx}(f \circ g) = \frac{d}{dx}(f(g)) = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx}(f(g(x)))$$

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

3.4 derivatives

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$$\frac{d}{dx}\left((2x+1)^3\right) =$$

$g(x)$ $f(x)$

$$x \longmapsto 2x+1 \longmapsto (\quad)^3$$

inside function — outside function

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx}(\text{above}) = 3(\,g(x)\,)^2 \cdot g'(x)$$

$$= 3(\,2x+1\,)^2 \cdot (2)$$

$\frac{d}{dx}(c) = 0$

$\frac{d}{dx}(x) = 1$

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

3.4 derivatives

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$$\frac{d}{dx}\left((x^3-2x+17)^{52} \right)$$

$$x \longmapsto \underset{g(x)}{(x^3-2x+17)} \longmapsto \overset{f(x)}{(\quad)^{52}}$$

$$\frac{d}{dx}(\text{above}) = 52(x^3-2x+17)^{51} \cdot g'(x)$$

$$= 52(x^3-2x+17)^{51}(3x^2-2)$$

SEGUE

$$52^{51}(x^3-2x+17)^{51} = (52(x^3-2x+17))^{51}$$

$$\frac{d}{dx}(c)=0$$

$$\frac{d}{dx}(x)=1$$

$$\frac{d}{dx}(x^n)=nx^{n-1}$$

$$\frac{d}{dx}(\sin x)=\cos x$$

$$\frac{d}{dx}(\cos x)=-\sin x$$

$$\frac{d}{dx}(\tan x)=\sec^2 x$$

$$\frac{d}{dx}(\sec x)=\sec x\tan x$$

$$\frac{d}{dx}(\cot x)=-\csc^2 x$$

$$\frac{d}{dx}(\csc x)=-\csc x\cot x$$

3.4 derivatives

2014-10-17 day 37

$$\star = \frac{d}{dx}\left(\sin^4(x^2+1) \right)$$

$$= \frac{d}{dx}\left([\sin(x^2+1)]^4 \right)$$

$$x \longmapsto x^2+1 \longmapsto \sin(\ ) \longmapsto (\ )^4$$

$$\frac{d}{dx}(\star) = 4(\sin(x^2+1))^3 \cdot \frac{d}{dx}(\sin(x^2+1))$$

$$x \mapsto x^2+1 \mapsto \sin(\ )$$

$$= 4(\sin(x^2+1))^3 \cdot \cos(x^2+1) \cdot \frac{d}{dx}(x^2+1)$$

$$= 4(\sin^3(x^2+1)) \cos(x^2+1)(2x)$$

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

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$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

3.4 derivatives

2014-10-17 day 37

$$\frac{d}{dx}\left(\sin^4(x^2+7)\right)$$

u^5 $5u^4$

$u=\sin(a)$ $\cos a$

$a=x^2+7$ $2x$

$5u^4 \cdot \cos(a) \cdot 2x$

$5(\sin(x^2+7))^4 \cdot \cos(x^2+7)$

$\cdot 2x$

$\frac{d}{dx}(c)=0$

$\frac{d}{dx}(x)=1$

$\frac{d}{dx}(x^n)=nx^{n-1}$

$\frac{d}{dx}(\sin x)=\cos x$

$\frac{d}{dx}(\cos x)=-\sin x$

$\frac{d}{dx}(\tan x)=\sec^2 x$

$\frac{d}{dx}(\sec x)=\sec x\tan x$

$\frac{d}{dx}(\cot x)=-\csc^2 x$

$\frac{d}{dx}(\csc x)=-\csc x\cot x$

3.4 derivatives

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I have $f(u)$

where $u = g(v)$

where $v = h(x)$

$$\frac{d}{dx} f = \frac{df}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$$

$\frac{d}{dx}(c) = 0$

$\frac{d}{dx}(x) = 1$

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

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3.4 derivatives

3.3/ 33-37, 39-40, 43-44, 45, 47, 49, 51, 57-60, 69, 71, 75-76, 80

3.4/ 25-27, 31-33, 36, 43

$\frac{d}{dx}(c)=0$

$\frac{d}{dx}(x)=1$

$\frac{d}{dx}(x^n)=nx^{n-1}$

$\frac{d}{dx}(\sin x)=\cos x$

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$\frac{d}{dx}(\csc x)=-\csc x\cot x$

3.4 derivatives

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3.4/17

$$f(x) = \frac{\sin x \sec x}{1 + x\tan x}$$

$\sin x\left(\frac{1}{\cos x}\right) = \tan x$

$\frac{d}{dx}(c) = 0$

$\frac{d}{dx}(x) = 1$

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

$$f'(x) = \frac{\frac{d}{dx}(\sin x \sec x)\cdot(1 + x\tan x) - (\sin x \sec x)\left(\frac{d}{dx}(1 + x\tan x)\right)}{(1 + x\tan x)^2}$$

$$= \frac{\left[\frac{d}{dx}(\sin x)\cdot\sec x + \sin x\cdot\frac{d}{dx}(\sec x)\right](1 + x\tan x) - (\sin x \sec x)\left[\frac{d}{dx}(x)\cdot\tan x + x\frac{d}{dx}(\tan x)\right]}{(1 + x\tan x)^2}$$

$$= \frac{\left[\cos x\cdot\sec x + \sin x \sec x \tan x\right](1 + x\tan x) - \sin x \sec x\left[\tan x + x\sec^2 x\right]}{(1 + x\tan x)^2}$$

$$= \frac{\left[1 + \tan^2 x\right](1 + x\tan x) - \tan x(\tan x + x\sec^2 x)}{(1 + x\tan x)^2}$$

$1 + \tan^2 x = \sec^2 x$

$\sin^2 + \cos^2 = 1$

$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$

$\tan^2 + 1 = \sec^2$

$-\tan^2 \quad -\tan^2$

$\sec^2 - \tan^2 = 1$

$$= \frac{\sec^2 x + x\sec^2 x\tan x - \tan^2 x - x\sec^2 x\tan x}{(1 + x\tan x)^2}$$

$$= \frac{1}{(1 + x\tan x)^2}$$

3.4 derivatives

2014-10-17 day 37

17alt) $f(x) = \dfrac{\tan x}{1+x\tan x}$

$$f'(x) = \frac{\frac{d}{dx}(\tan x)(1+x\tan x) - \tan x \frac{d}{dx}(1+x\tan x)}{(1+x\tan x)^2}$$

$$= \frac{\sec^2 x(1+x\tan x) - \tan x\left[1(\tan x) + x\sec^2 x\right]}{(1+x\tan x)^2}$$

$$= \frac{\sec^2 x + x\sec^2 x\tan x - \tan^2 x - x\sec^2 x\tan x}{(1+x\tan x)^2}$$

$\frac{d}{dx}(c) = 0$

$\frac{d}{dx}(x) = 1$

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(\sin x) = \cos x$

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3.4 derivatives

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